MATH 1650: SECTION 9.1: SEQUENCES

Informally, we can think of a sequence as an infinite list of numbers. For example, consider the sequence

$$\frac{1}{2}, -\frac{3}{4}, \frac{9}{8}, -\frac{27}{16}, \ldots$$

As usual, the periods of ellipsis, ..., indicate that the proposed pattern continues forever. Each of the numbers in the list is called a *term*, and we call $\frac{1}{2}$ the 'first term', $-\frac{3}{4}$ the 'second term', $\frac{9}{8}$ the 'third term' and so forth. In numbering them this way, we are setting up a function, which we'll call 'a' per tradition, between the natural numbers and the terms in the sequence.

n	a_n
1	$\frac{1}{2}$
2	$-\frac{3}{4}$
3	$\frac{9}{8}$
4	$-\frac{27}{16}$
$\vdots$	$\vdots$

In other words, a_n is the n^{th} term in the sequence: $a_1 = \frac{1}{2}$, $a_2 = -\frac{3}{4}$, $a_3 = \frac{9}{8}$, $a_4 = -\frac{27}{16}$ and so on.

EXAMPLE: Write the first four terms of the following sequences.

1. $a_n = \dfrac{5^{n-1}}{3^n}$, $n \geq 1$

2. $b_k = \dfrac{(-1)^k}{2k+1}$, $k \geq 0$

3. $\{2n-1\}_{n=1}^{\infty}$

4. $\left\{\dfrac{1+(-1)^i}{i}\right\}_{i=2}^{\infty}$

EXAMPLE: Write out the first four terms of the sequences defined recursively:

5. $a_1 = 7,\ a_{n+1} = 2 - a_n,\ n \geq 1$

6. $f_0 = 1,\ f_n = n \cdot f_{n-1},\ n \geq 1$

ARITHMETIC AND GEOMETRIC SEQUENCES: Suppose $\{a_n\}$ is a sequence.

- If there is a number d so that $a_{n+1} = a_n + d$ for all n, then $\{a_n\}$ is called an **arithmetic sequence**. The number d is called the **common difference**.

  NOTE: In an arithmetic sequence, $a_{n+1} - a_n = d$ for all n which is why 'd' is called the 'common' difference.

- If there is a number r so that $a_{n+1} = ra_n$ for all n, then $\{a_n\}$ is called a **geometric sequence**. The number r is called the **common ratio**.

  NOTE: In a geometric sequence, $\dfrac{a_{n+1}}{a_n} = r$ for all n which is why 'r' is called the 'common' ratio.

EXAMPLE: Determine if the following sequences are arithmetic, geometric or neither. If arithmetic, find the common difference d; if geometric, find the common ratio r.

7. $a_n = \dfrac{5^{n-1}}{3^n},\ n \geq 1$

8. $b_k = \dfrac{(-1)^k}{2k+1},\ k \geq 0$

9. $\{2n-1\}_{n=1}^{\infty}$

10. $\dfrac{1}{2}, -\dfrac{3}{4}, \dfrac{9}{8}, -\dfrac{27}{16}, \ldots$

FORMULAS FOR ARITHMETIC AND GEOMETRIC SEQUENCES:

- An arithmetic sequence with first term $a_1 = a$ and common difference d is given by

$$a_n = a + (n-1)d, \quad n \geq 1$$

  NOTE: Arithmetic sequences are linear functions with slope d.

- A geometric sequence with first term $a_1 = a$ and common ratio $r \neq 0$ is given by

$$a_n = ar^{n-1}, \quad n \geq 1$$

  NOTE: Geometric sequences are exponential functions with base r.

EXAMPLE: Find an explicit formula for the n^{th} term of the following sequences.

11. $0.9, 0.09, 0.009, 0.0009, \ldots$

12. $\frac{2}{5}, 2, -\frac{2}{3}, -\frac{2}{7}, \ldots$

    HINT: Write $2 = \frac{2}{1}$ and look for a pattern in the denominator.

13. $1, -\frac{2}{7}, \frac{4}{13}, -\frac{8}{19}, \ldots$

    HINT: Write $1 = \frac{1}{1}$ and look for separate patterns in the numerator and denominator.

HOMEWORK: Section 9.1: 1 - 35 odd.